

# A General Approach to Underapproximate Reasoning about Concurrent Programs

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# State of the Art: **Correctness**

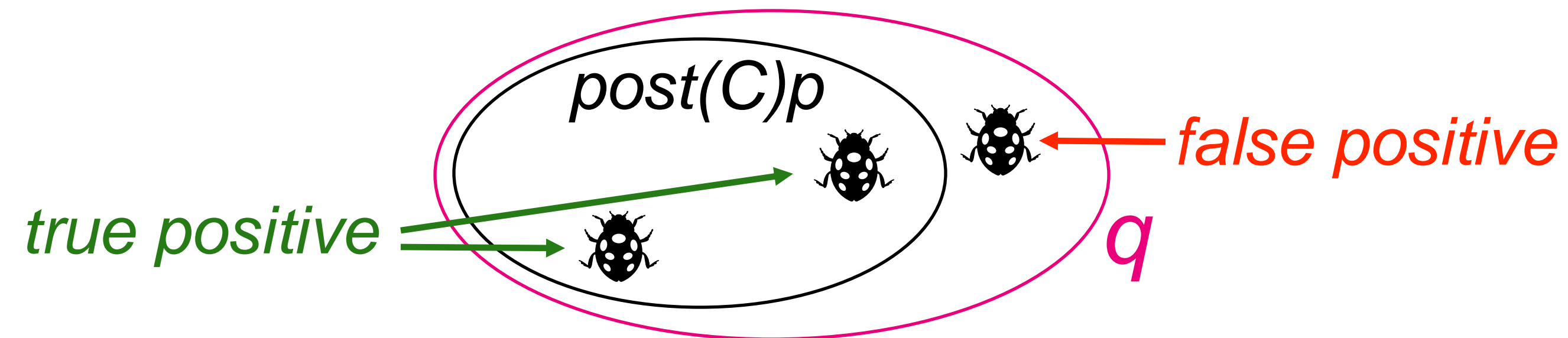
- ❖ Lots of work on *reasoning* for proving *correctness*
  - Prove the *absence of bugs*
  - *Overapproximate* reasoning
  - **Compositionality**
    - in *code*  $\Rightarrow$  reasoning about *incomplete components*
    - in *resources* accessed  $\Rightarrow$  spatial locality
  - **Scalability** to large teams and codebases

# Hoare Logic (HL)

Hoare triples  $\{p\} C \{q\}$  iff  $\text{post}(C)p \subseteq q$

$q$  overapproximates  $\text{post}(C)p$

For all states  $s$  in  $p$   
if running  $C$  on  $s$  terminates in  $s'$ , then  $s'$  is in  $q$





***Incorrectness Logic:***  
A Formal Foundation  
for  
Bug Catching

# Incorrectness Logic (IL)

Hoare triples  $\{p\} C \{q\}$  *iff*  $\text{post}(C)p \subseteq q$

*For all states  $s$  in  $p$   
if running  $C$  on  $s$  terminates in  $s'$ , then  $s'$  is in  $q$*

Incorrectness  
triples  $[p] C [q]$  *iff*  $\text{post}(C)p \not\subseteq q$

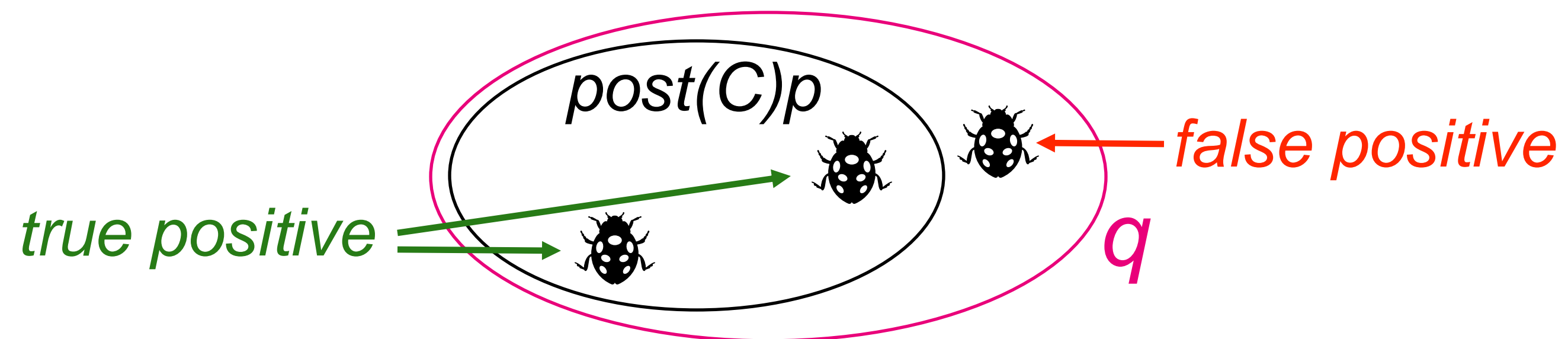
*For all states  $s$  in  $q$   
 $s$  can be reached by running  $C$  on some  $s'$  in  $p$*

# Incorrectness Logic (IL)

Hoare triples

$$\{p\} C \{q\} \quad \text{iff} \quad \text{post}(C)p \subseteq q$$

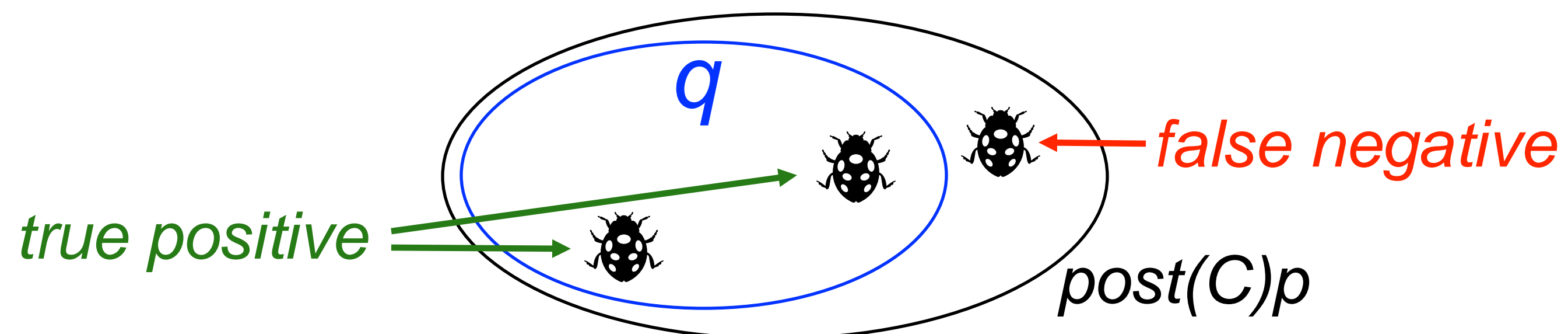
$q$  *overapproximates*  $\text{post}(C)p$



Incorrectness triples

$$[p] C [q] \quad \text{iff} \quad \text{post}(C)p \supseteq q$$

$q$  *underapproximates*  $\text{post}(C)p$



# Incorrectness Logic (IL)

$$[p] C [\varepsilon: q]$$

$\varepsilon$ : exit condition

ok: normal execution

er: erroneous execution

$$[y=v] x:=y [ok: x=y=v]$$
$$[p] \text{error}(\ ) [er: p]$$

# IL Proof Rules and Principles (Sequencing)

$$\frac{[p] C_1 [er: q]}{[p] C_1; C_2 [er: q]}$$

$$\frac{[p] C_1 [ok: r] \quad [r] C_2 [\varepsilon: q]}{[p] C_1; C_2 [\varepsilon: q]}$$

- ❖ Short-circuiting semantics for errors



# IL Proof Rules and Principles (Branches)

$$\frac{[p] C_i [\varepsilon: q] \quad \text{some } i \in \{1, 2\}}{[p] C_1 + C_2 [\varepsilon: q]}$$

- ❖ Drop paths/branches (this is a sound underapproximation)
- ❖ Scalable bug detection!

# Compositionality: Incorrectness Separation Logic (ISL)

IL

$$[p] \text{ C } [\varepsilon: q]$$

SL

$$\frac{\{p\} \text{ C } \{q\}}{\{p * r\} \text{ C } \{q * r\}}$$

$x \mapsto - * x \mapsto - \Leftrightarrow \text{false}$   
 $x \mapsto v * \text{emp} \Leftrightarrow x \mapsto v$

ISL

$$\frac{[p] \text{ C } [\varepsilon: q]}{[p * r] \text{ C } [\varepsilon: q * r]}$$

# Concurrency: Concurrent Incorrectness Separation Logic (CISL)

ISL

$$\frac{[p] \text{ C } [\varepsilon: q]}{[p * r] \text{ C } [\varepsilon: q * r]}$$

CSL

$$\frac{\{p_1\} \text{ C}_1 \{q_1\} \quad \{p_2\} \text{ C}_2 \{q_2\}}{\{p_1 * p_2\} \text{ C}_1 \parallel \text{ C}_2 \{q_1 * q_2\}}$$

CISL

$$\frac{[p_1] \text{ C}_1 [\varepsilon: q_1] \quad [p_2] \text{ C}_2 [\varepsilon: q_2]}{[p_1 * p_2] \text{ C}_1 \parallel \text{ C}_2 [\varepsilon: q_1 * q_2]}$$

# Global Concurrency Bugs

Due to two or more threads, under certain interleavings:

1. *data-agnostic*: threads do not affect one another's control flow

$$\begin{array}{l} \text{free}(x); \quad \parallel \quad a := [z]; \\ [z] := 1; \quad \parallel \quad \text{if } (*) \text{ L: } [x] := 1 \\ \text{data-agnostic use-after-free bug at L} \end{array}$$

## CISL

2. *data-dependent* bugs: threads do affect one another's control flow

$$\begin{array}{l} \text{free}(x); \quad \parallel \quad a := [z]; \\ [z] := 1; \quad \parallel \quad \text{if } (a=1) \text{ L: } [x] := 1 \\ \text{data-dependent use-after-free bug at L} \end{array}$$

not handled compositionally in CISL theory

# Concurrent Adversarial Separation Logic (CASL)

CISL

$$\frac{[p_1] C_1 [\varepsilon: q_1] \quad [p_2] C_2 [\varepsilon: q_2]}{[p_1 * p_2] C_1 \parallel C_2 [\varepsilon: q_1 * q_2]}$$

Rely-Guarantee

$$\frac{R_1, G_1 \vdash \{p\} C_1 \{q\} \quad R_2, G_2 \vdash \{p\} C_2 \{q\}}{R_1 \cap R_2, G_1 \cup G_2 \vdash \{p\} C_1 \parallel C_2 \{q\}}$$

CASL

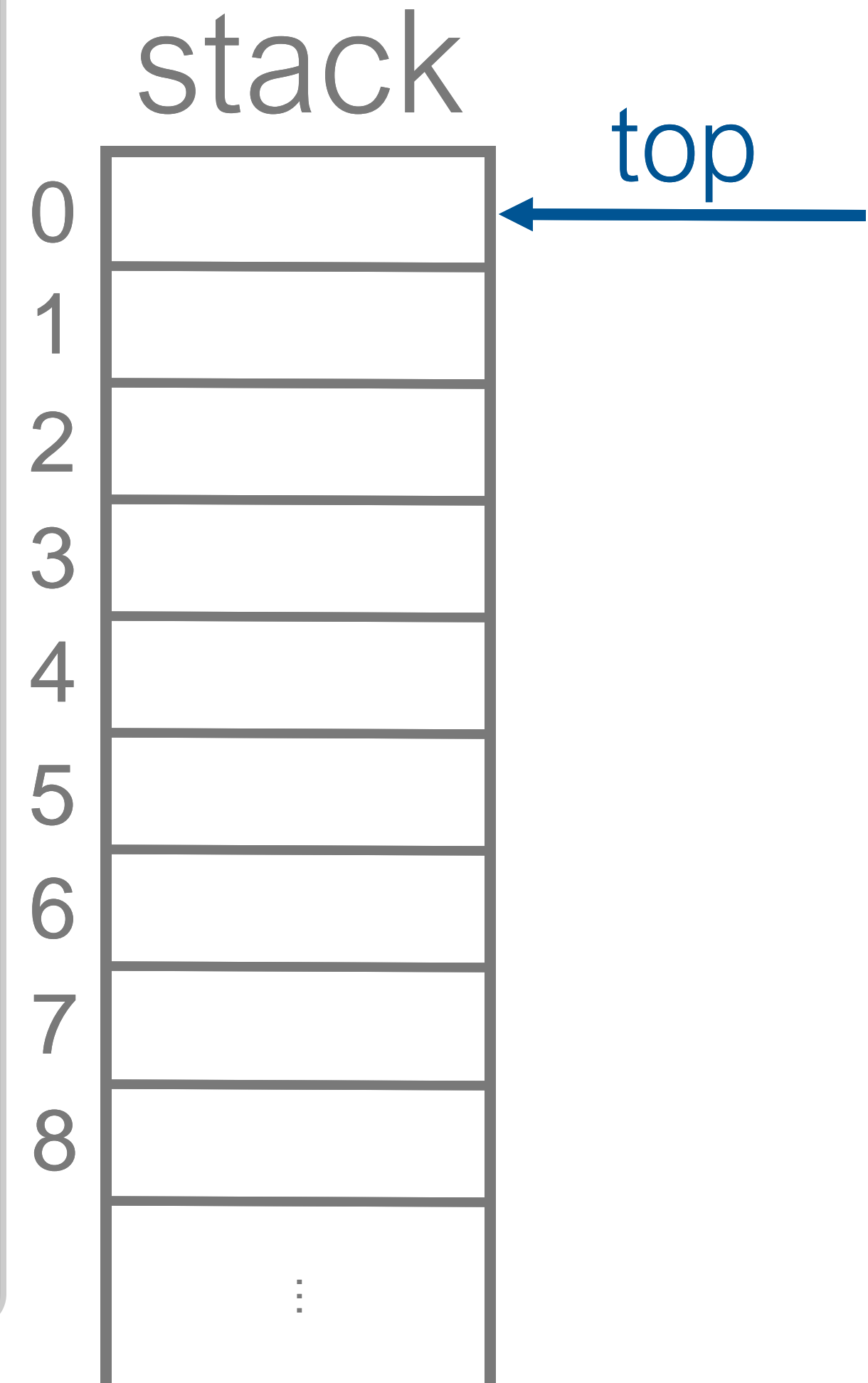
$$\frac{R_1, G_1, \Theta \vdash [p_1] C_1 [\varepsilon: q_1] \quad R_2, G_2, \Theta \vdash [p_2] C_2 [\varepsilon: q_2]}{R_1 \cap R_2, G_1 \cup G_2, \Theta \vdash [p_1 * p_2] C_1 \parallel C_2 [\varepsilon: q_1 * q_2]}$$

# CASL: Information Disclosure Attacks

$c \mapsto []$

```
send(c, 8);  
recv(c, y);
```

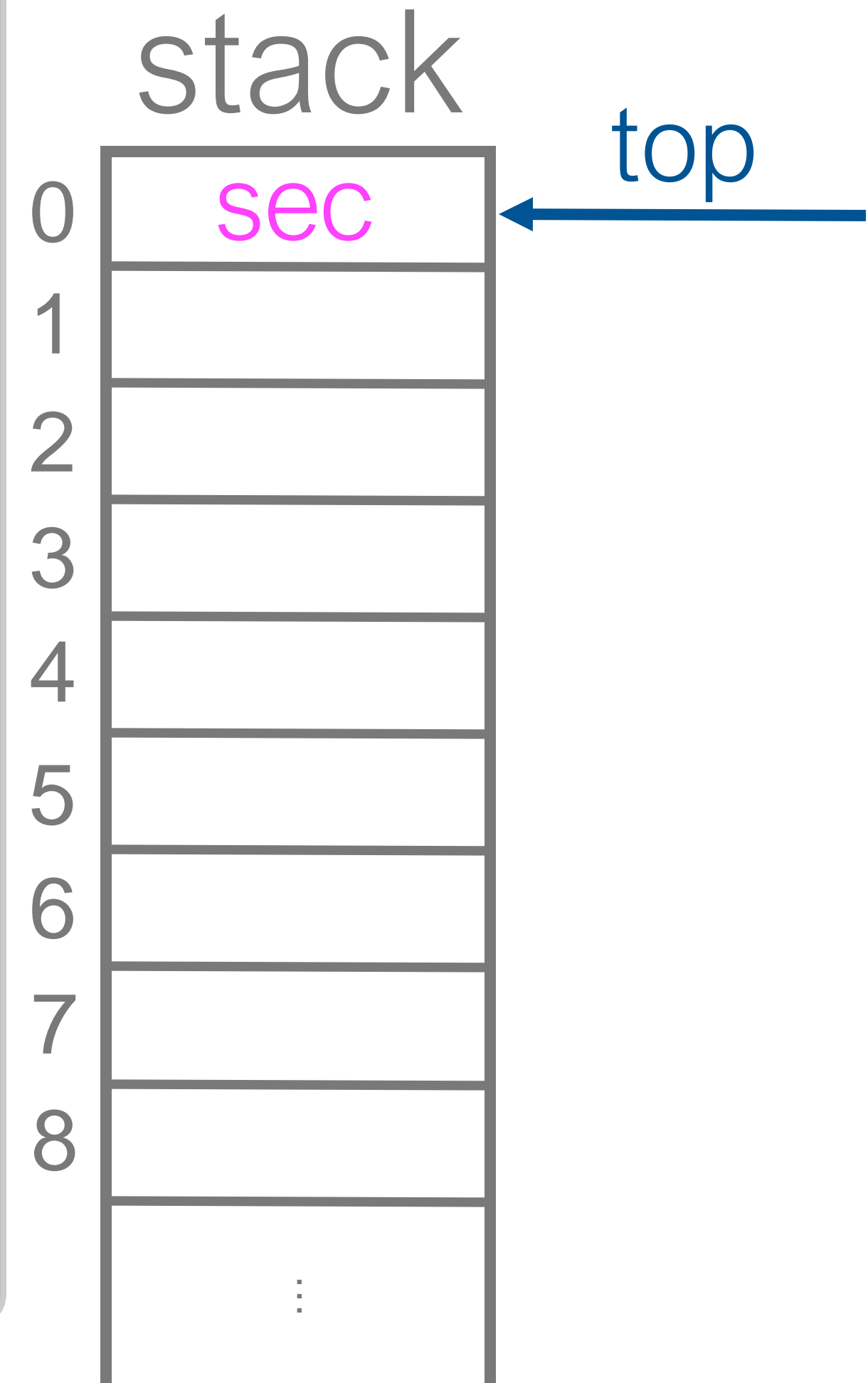
```
local sec := *;  
local w[8] := {0};  
recv(c, x);  
if (x ≤ 8)  
    z := w[x];  
    send(c, z);
```



# CASL: Information Disclosure Attacks

```
send(c, 8);  
recv(c, y);
```

```
    c ↦ []  
    local sec := *;  
    sec = top * c ↦ []  
    local w[8] := {0};  
    recv(c, x);  
    if (x ≤ 8)  
        z := w[x];  
        send(c, z);
```

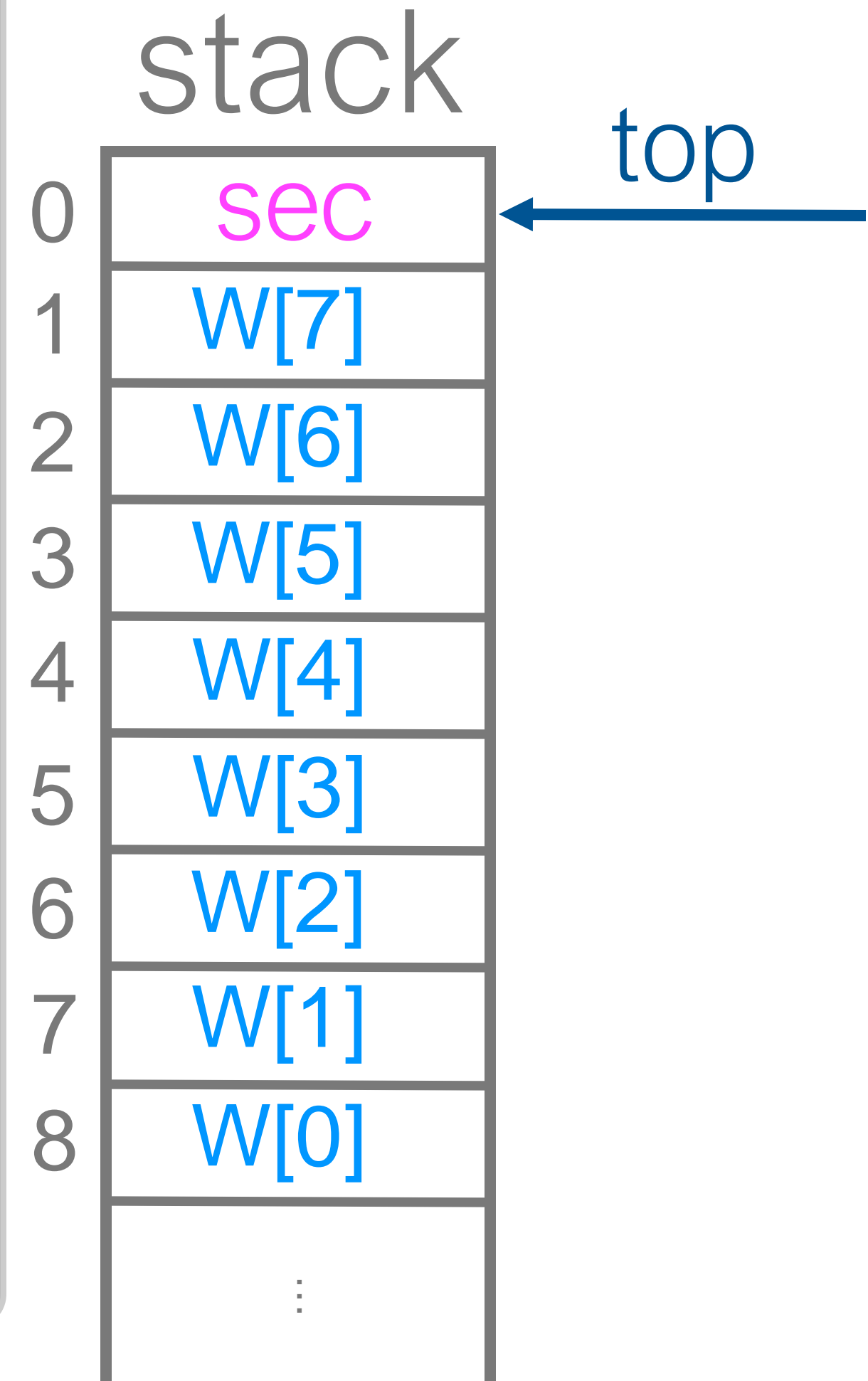


# CASL: Information Disclosure Attacks

```
send(c, 8);  
recv(c, y);
```

$c \mapsto []$

```
local sec := *;  
local w[8] := {0};  
sec=top * w[0]=top-8 * c  $\mapsto []$   
recv(c, x);  
if (x ≤ 8)  
    z := w[x];  
    send(c, z);
```





# CASL: Information Disclosure Attacks

```
send(c, 8);  
recv(c, y);
```

```
 $c \mapsto []$   
local sec := *;  
local w[8] := {0};  
sec=top * w[8]=sec *  $c \mapsto []$   
recv(c, x);  
if (x ≤ 8)  
    z := w[x];  
    send(c, z);
```



$G_a(\alpha_1): c \mapsto [] \rightsquigarrow c \mapsto [(8, \tau_a, 0)]$

$R_v = G_a$

# CASL: Information Disclosure Attacks

$c \mapsto []$

```
send(c, 8);
recv(c, y);
```

```
local sec := *;
local w[8] := {0}; //  $R_v(\alpha_1)$ 
sec=top * w[8]=sec *  $c \mapsto [(8, \tau_a, 0)]$ 
recv(c, x);
if (x ≤ 8)
    z := w[x];
    send(c, z);
```



$G_a(\alpha_1): c \mapsto [] \rightsquigarrow c \mapsto [(8, \tau_a, 0)]$

$R_v = G_a$

$G_v(\alpha_2): c \mapsto [(8, \tau_a, 0)] \rightsquigarrow c \mapsto []$

# CASL: Information Disclosure Attacks

```
send(c, 8);
recv(c, y);
```

$c \mapsto []$

```
local sec := *;
local w[8] := {0};
recv(c, x); //  $G_v(\alpha_2)$ 
```

```
sec=top * w[8]=sec * x=(8,...) *  $c \mapsto []$ 
if (x ≤ 8)
    z := w[x];
    send(c, z);
```



$G_a(\alpha_1): c \mapsto [] \rightsquigarrow c \mapsto [(8, \tau_a, 0)]$

$R_v = G_a$

$G_v(\alpha_2): c \mapsto [(8, \tau_a, 0)] \rightsquigarrow c \mapsto []$

# CASL: Information Disclosure Attacks

```

send(c, 8);
recv(c, y);

```

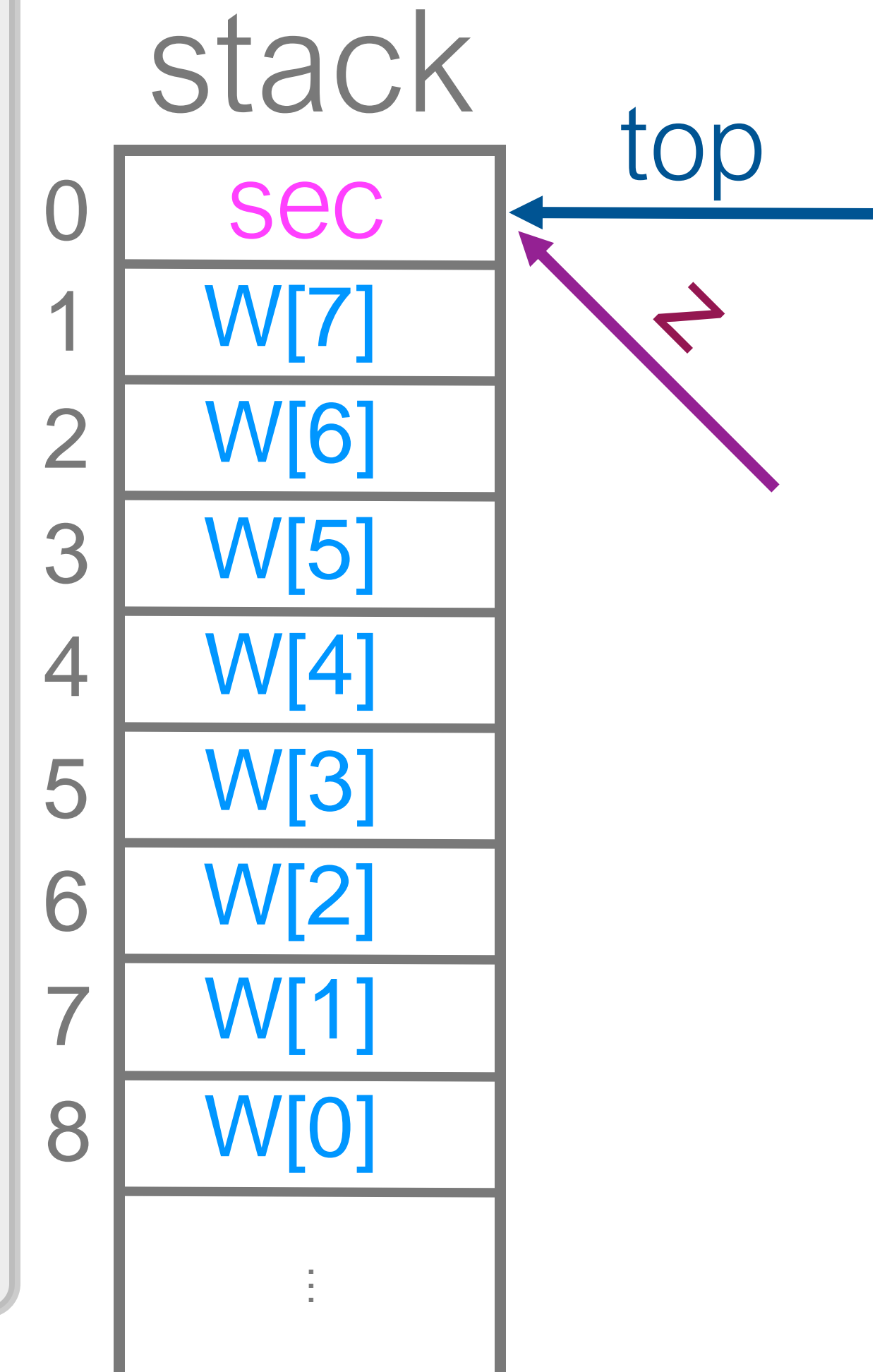
$c \mapsto []$

```

local sec := *;
local w[8] := {0};
recv(c, x);
if (x ≤ 8)
    z := w[x];
sec=top * w[8]=sec * z=sec * c ↦ []
send(c, z);

```

$sec=top * w[8]=sec * z=sec * c \mapsto []$



$G_a(\alpha_1): c \mapsto [] \rightsquigarrow c \mapsto [(8, \tau_a, 0)]$

$R_v = G_a$

$G_v(\alpha_2): c \mapsto [(8, \tau_a, 0)] \rightsquigarrow c \mapsto []$

$G_v(\alpha_3): c \mapsto [] \rightsquigarrow c \mapsto [(-, \tau_v, 1)]$

# CASL: Information Disclosure Attacks

```

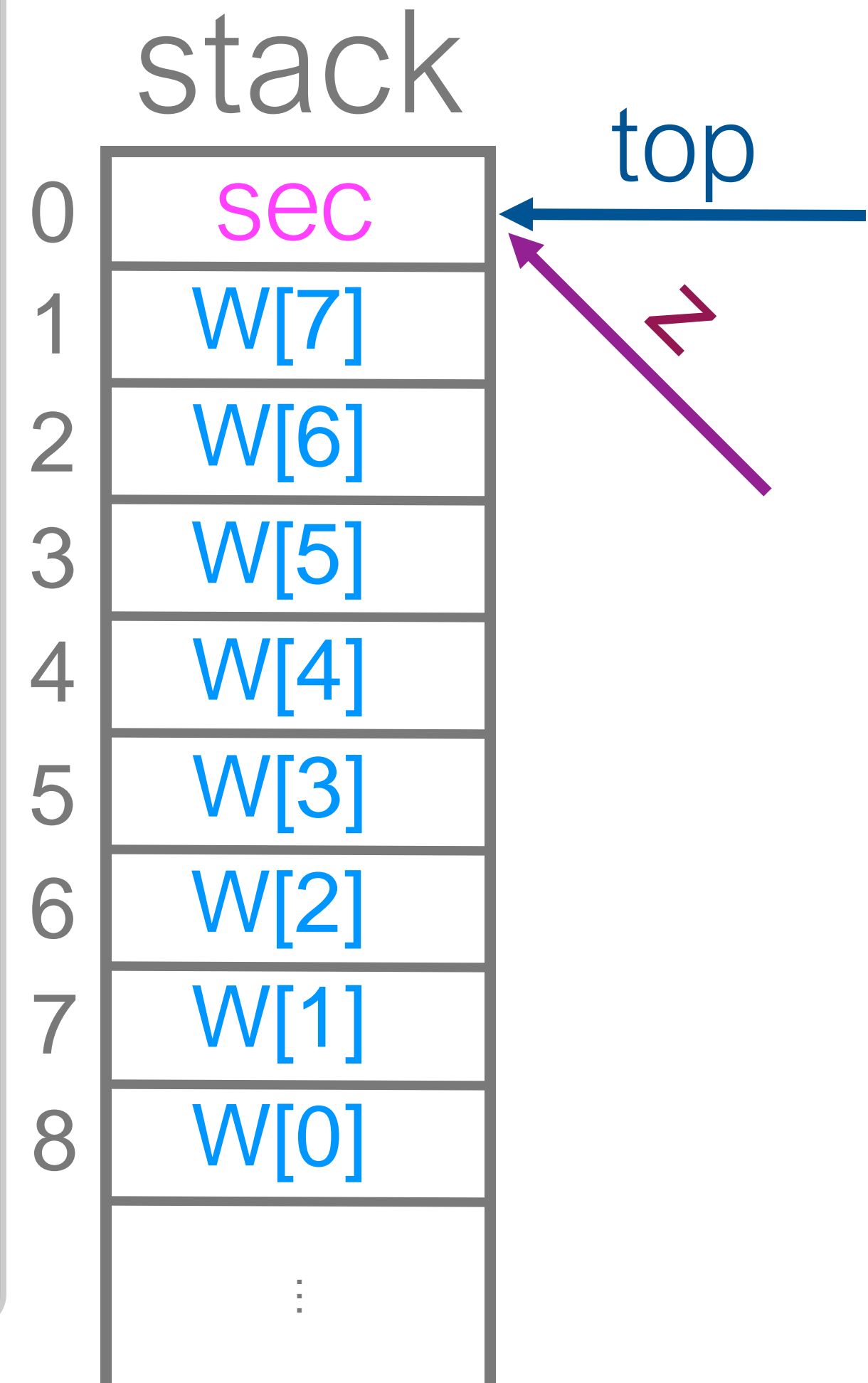
send(c, 8);
recv(c, y);

```

```

c ↦ [ ]
local sec := *;
local w[8] := {0};
recv(c, x);
if (x ≤ 8)
  z := w[x];
  send(c, z); // Gv(α3)
z = sec ↦ [(sec, τv, 1)] // Ry(α4)
error: information disclosure!

```



$G_a(\alpha_1): c \mapsto [ ] \rightsquigarrow c \mapsto [(8, \tau_a, 0)]$

$R_v = G_a$

$G_a(\alpha_4): c \mapsto [(v, \tau_v, 1)] \rightsquigarrow c \mapsto [ ]$

$G_v(\alpha_2): c \mapsto [(8, \tau_a, 0)] \rightsquigarrow c \mapsto [ ]$

$G_v(\alpha_3): c \mapsto [ ] \rightsquigarrow c \mapsto [(-, \tau_v, 1)]$

# CASL: Information Disclosure Attacks

$c \mapsto []$

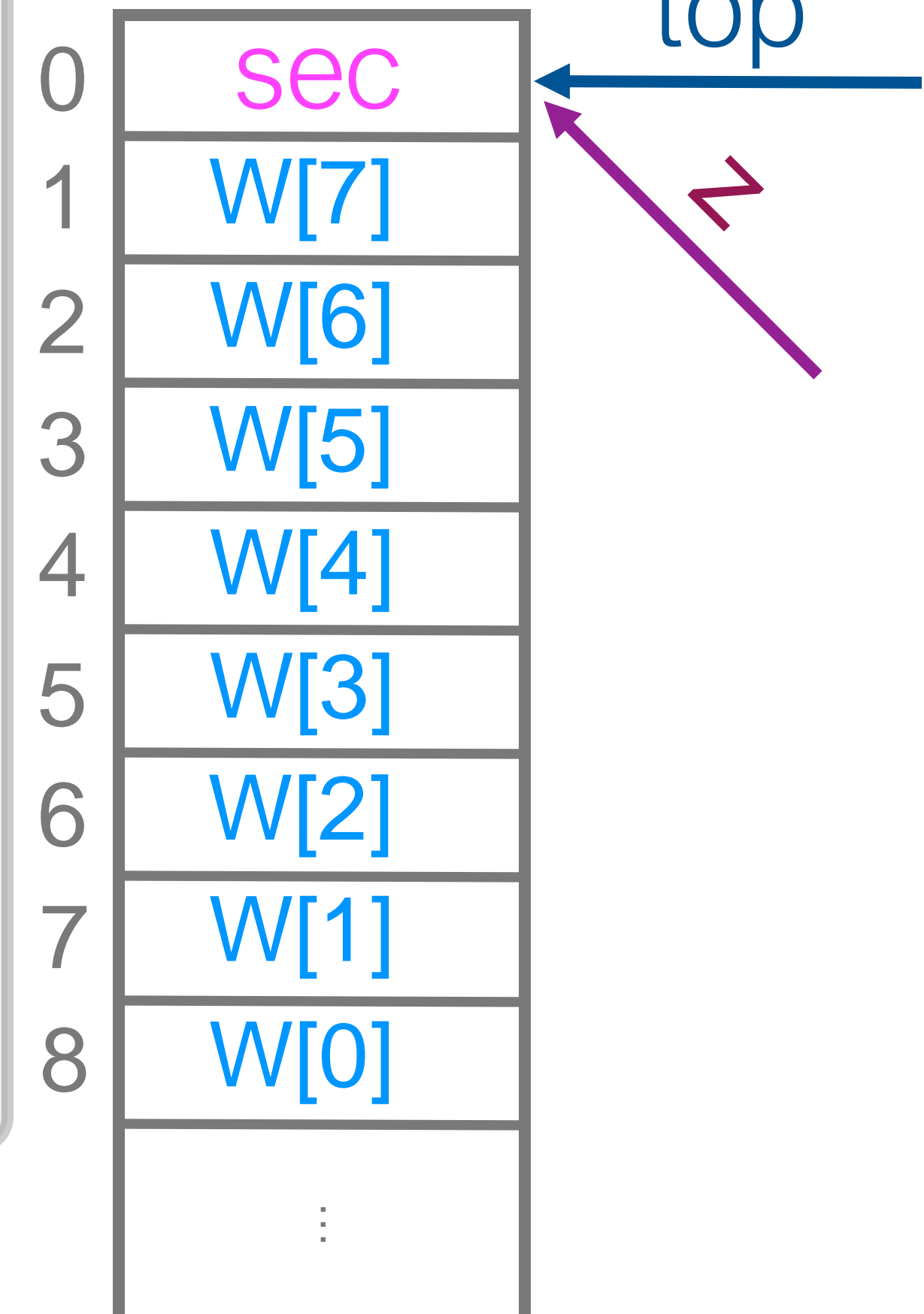
```
send(c, 8);
recv(c, y);
```

$c \mapsto []$

```
local sec := *;
local w[8] := {0};
recv(c, x);
if (x ≤ 8)
  z := w[x];
  send(c, z);
```

$c \mapsto [(sec, \tau_v, 1)]$

stack



$G_a(\alpha_1): c \mapsto [] \rightsquigarrow c \mapsto [(8, \tau_a, 0)]$

$R_v = G_a$

$G_a(\alpha_4): c \mapsto [(v, \tau_v, 1)] \rightsquigarrow c \mapsto []$

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$R_a = G_v$

$G_v(\alpha_3): c \mapsto [] \rightsquigarrow c \mapsto [(-, \tau_v, 1)]$

# CASL: Information Disclosure Attacks

$c \mapsto []$

`send(c, 8); //  $G_a(\alpha_1)$`

$c \mapsto [(8, \tau_a, 0)]$

`recv(c, y);`

$c \mapsto []$

`local sec := *;`

`local w[8] := {0};`

`recv(c, x);`

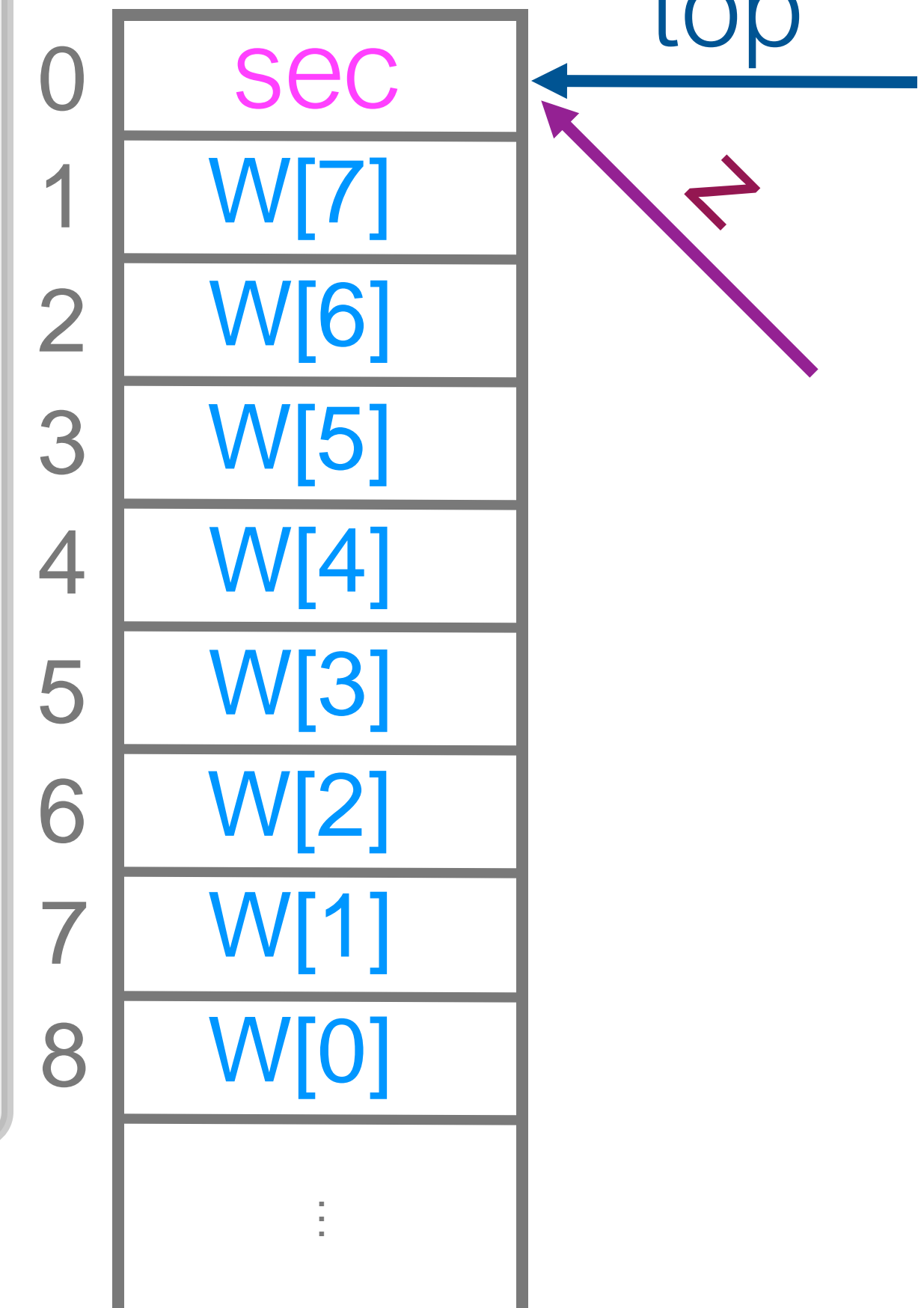
`if (x ≤ 8)`

`z := w[x];`

`send(c, z);`

$c \mapsto [(sec, \tau_v, 1)]$

stack



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$G_v(\alpha_3): c \mapsto [] \rightsquigarrow c \mapsto [(-, \tau_v, 1)]$

# CASL: Information Disclosure Attacks

$c \mapsto []$

`send(c, 8); //  $G_a(\alpha_1)$`

$c \mapsto [(8, \tau_a, 0)]$

`//  $R_a(\alpha_2)$ ;`

$c \mapsto []$

`recv(c, y);`

$c \mapsto []$

`local sec := *;`

`local w[8] := {0};`

`recv(c, x);`

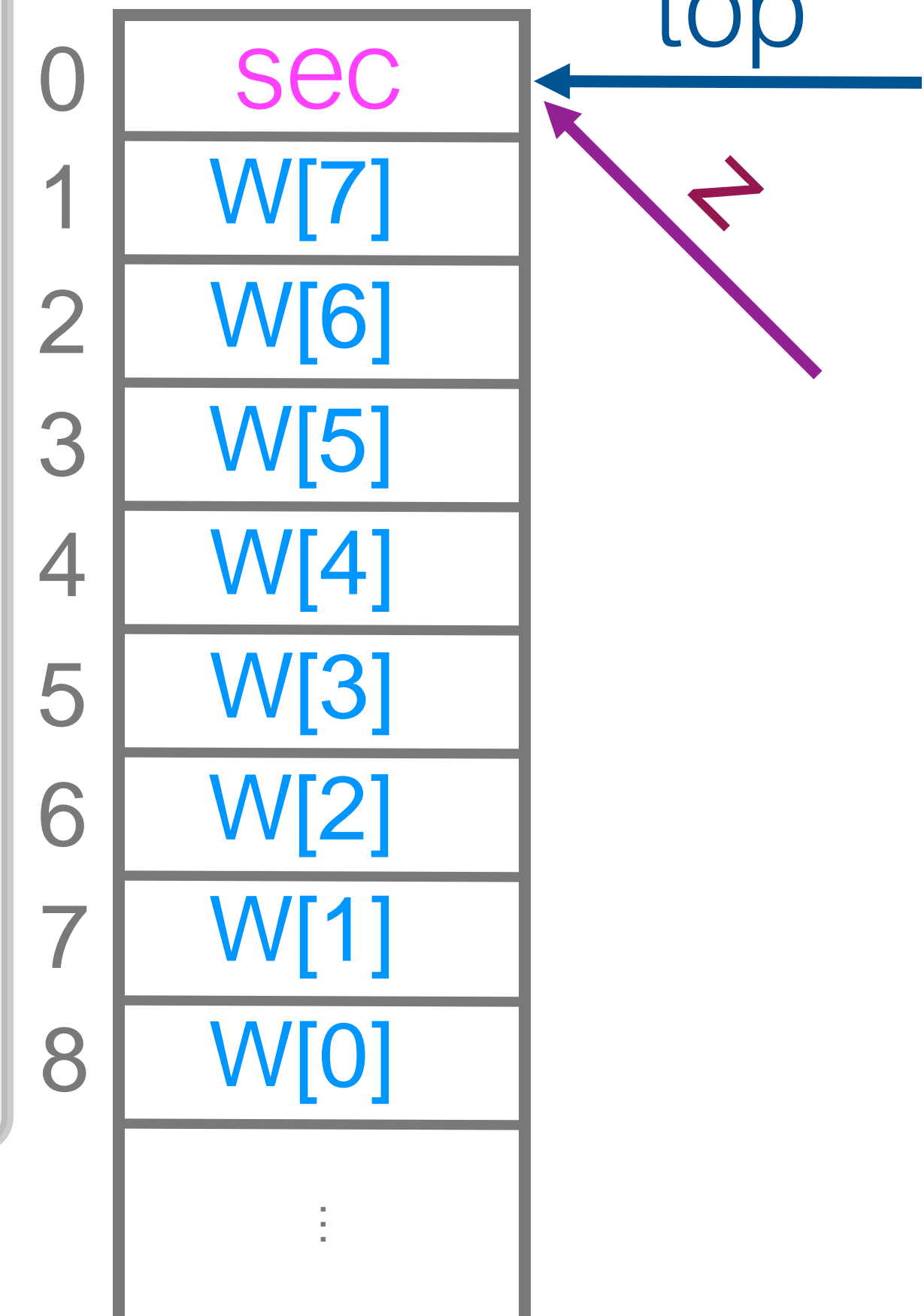
`if (x ≤ 8)`

`z := w[x];`

`send(c, z);`

$c \mapsto [(sec, \tau_v, 1)]$

stack



$G_a(\alpha_1): c \mapsto [] \rightsquigarrow c \mapsto [(8, \tau_a, 0)]$

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$G_v(\alpha_3): c \mapsto [] \rightsquigarrow c \mapsto [(-, \tau_v, 1)]$



# CASL: Information Disclosure Attacks

$c \mapsto []$

`send(c, 8); //  $G_a(\alpha_1)$`

$c \mapsto [(8, \tau_a, 0)]$

//  $R_a(\alpha_2); R_a(\alpha_3)$

$c \mapsto [(v, \tau_v, 1)]$

`recv(c, y);`

$c \mapsto []$

`local sec := *;`

`local w[8] := {0};`

`recv(c, x);`

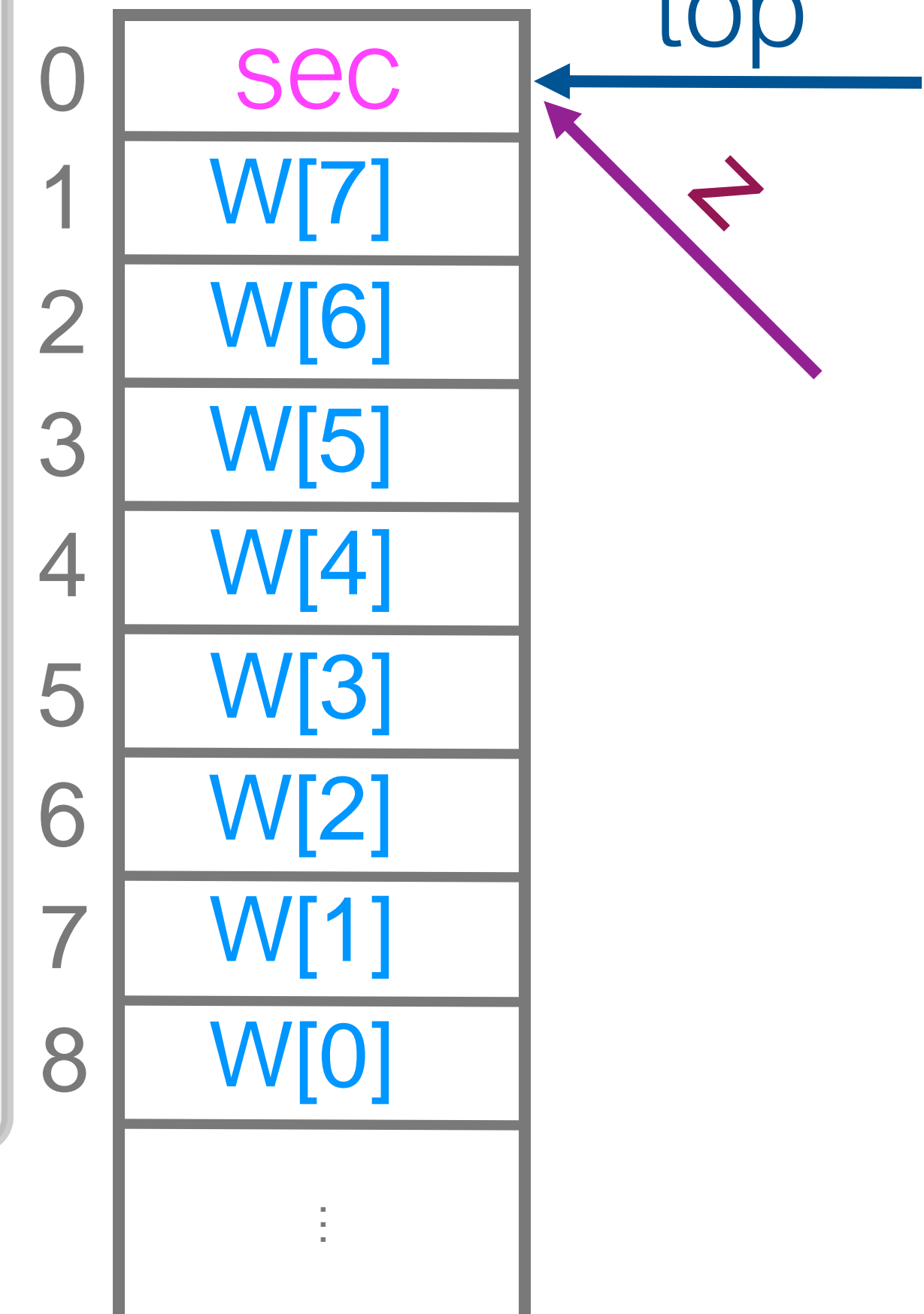
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$R_a = G_v$

$G_v(\alpha_3): c \mapsto [] \rightsquigarrow c \mapsto [(-, \tau_v, 1)]$

# CASL: Information Disclosure Attacks

$c \mapsto []$

`send(c, 8); //  $G_a(\alpha_1)$`

`//  $R_a(\alpha_2); R_a(\alpha_3)$`

$c \mapsto [(v, \tau_v, 1)]$

`recv(c, y); //  $G_a(\alpha_4)$`

error: information disclosure!

$c \mapsto []$

`local sec := *;`

`local w[8] := {0};`

`recv(c, x);`

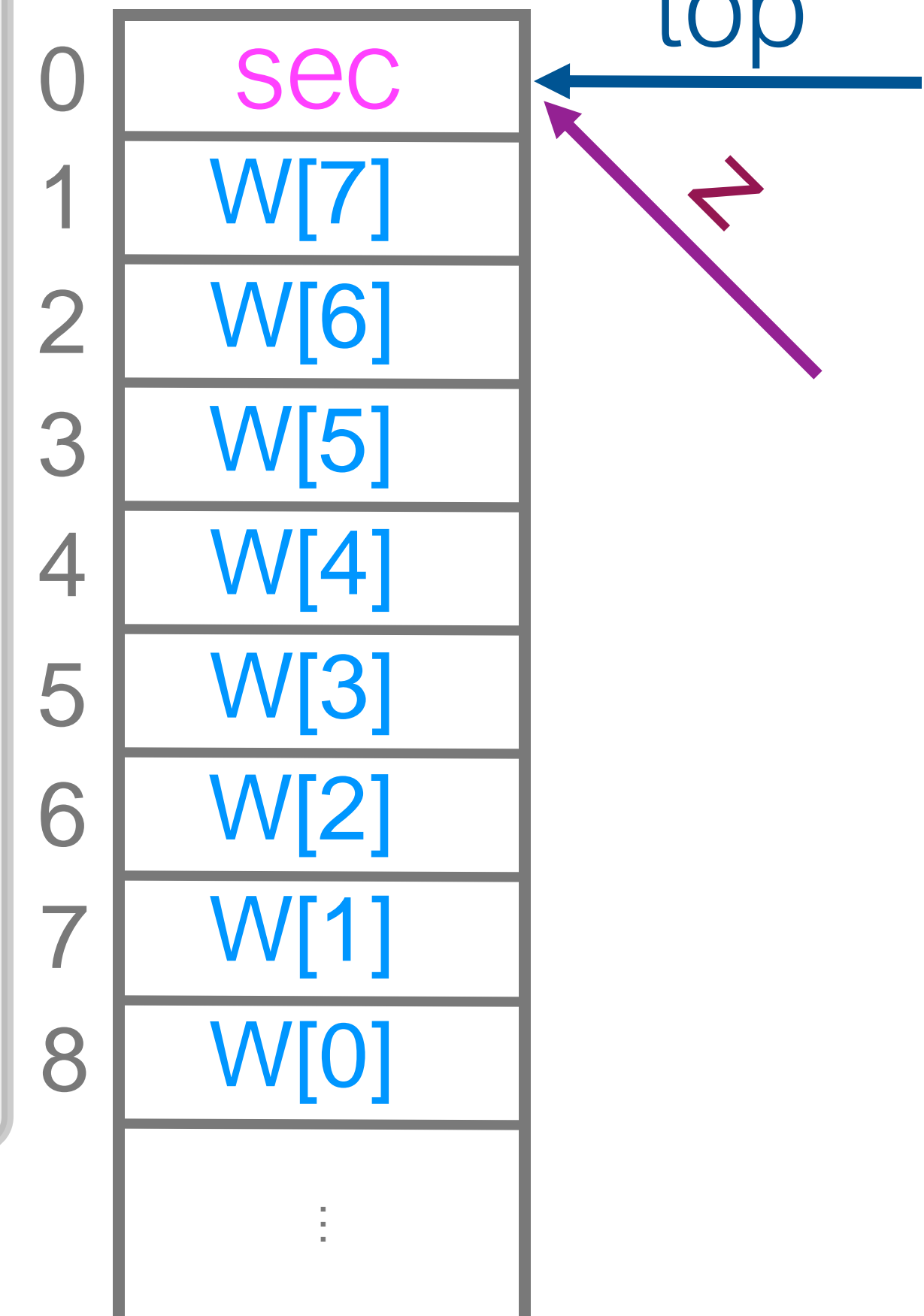
`if (x ≤ 8)`

`z := w[x];`

`send(c, z);`

$c \mapsto [(sec, \tau_v, 1)]$

stack



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# Conclusions

- ❖ CASL: *A general framework for concurrent underapproximate reasoning*
- ❖ It subsumes CISL
- ❖ Can handle *both* data-agnostic and data-dependent bugs *compositionally*
- ❖ Used to detect *software vulnerabilities leading to exploits/attacks*
  - 👉 model a vulnerable program  $C_v$  and its adversary  $C_a$  as  $C_a \parallel C_v$
- ❖ Instantiated for:
  - Information disclosure attacks (over stacks & heaps)
  - Buffer overflow attacks (over stacks & heaps)
  - Memory safety attacks (e.g., zero allocation)

Thank You for Listening!